TRINITY COLLEGE

Mathematics Specialist Units 3,4
Test 3 2019

Calculator Assumed

Vector Calculus

STUDENT'S NAME _____

DATE: Wednesday 15th May **TIME:** 55 minutes **MARKS**: 54

INSTRUCTIONS:

Standard Items: Pens, pencils, drawing templates, eraser

Special Items: Three calculators, notes on one side of a single A4 page (these notes to be handed in with this assessment)

Questions or parts of questions worth more than 2 marks require working to be shown to receive full marks.

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1. (12 marks)

Consider the following system of equations:

$$\begin{aligned} x+\ \ y+\ \ \ \ \ \ z &= 2 \\ x+2y+\ \ (k-5)z &= 2 \\ 3x+2y+(k^2+3)z &= k+9 \end{aligned}$$

(a) Represent this system as an augmented matrix and reduce it to row-echelon form. [3]

(b) Determine the value/s of k for which the system will have

(i) no solution [3]

(ii) a unique solution [2]

(iii) infinitely many solutions [2]

(c) For the value of k obtained in (b)(iii), explain why the system of equations has infinitely many solutions. [2]

2. (10 marks)

As part of a stunt in a movie, a car is driven off a cliff 80 metres high at a horizontal speed of 20 m/sec. Assume acceleration due to gravity is 9.8 m/sec^2. Determine each of the following using vector calculus:

(a) the velocity vector and the displacement vector [4]

(b) when the car hits the ground [2]

(c) how far from the edge of the cliff the car hits the ground [1]

(d) the speed of the car when it hits the ground [2]

(e) the angle of travel at the instant the car hits the ground [1]

3. (20 marks)

The position of a small body at any time t seconds is given by
$\mathbf{r}(t) = 24\sin\left(\frac{\pi t}{6}\right)\mathbf{i} + 24\cos\left(\frac{\pi t}{6}\right)\mathbf{j},\ t \geq 0$.

(a) Determine an expression for the velocity $v(t)$ of the body. [2]

(b) What is the speed of the body when $t = 4$, and what angle to the x-axis is the body moving at this instant? [4]

(c) Determine the distance of the body from (0,0) at any time t, and interpret this result in terms of the path described by the body. [3]

(d) Using the result of (c), determine the direction of movement of the body. [2]

(e) Determine $r(t) \bullet v(t)$ [2]

(f) Explain the significance of the answer to (e). [2]

(g) Determine $\int_0^2 v(t)dt$ and interpret the answer. [3]

(h) Explain why $\int_0^T |v(t)|dt > \int_0^T v(t)dt$ for all $T > 0$. [2]

4. (12 marks)

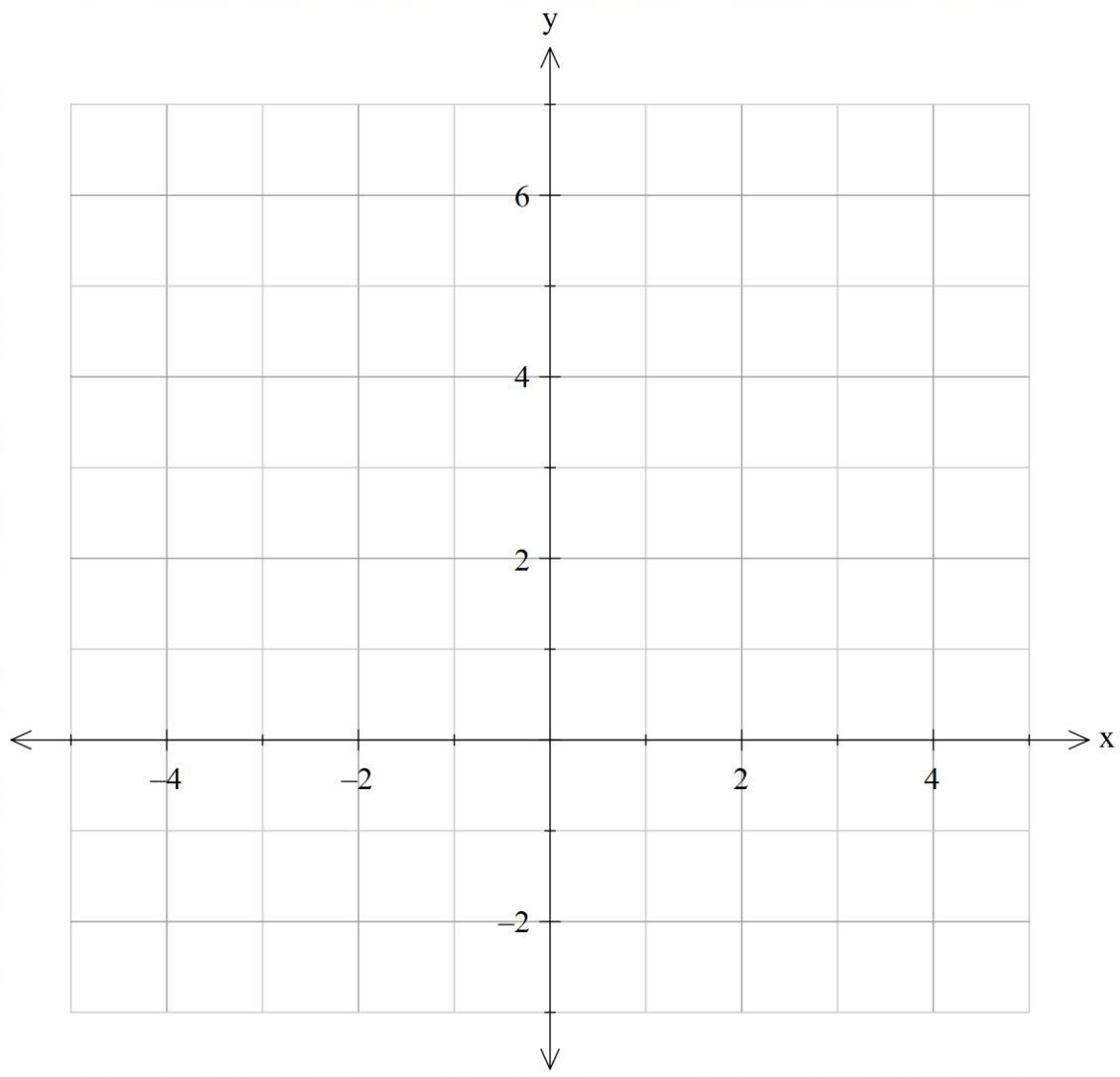
The acceleration of a particle at time t seconds is given by $a(t) = -4i + 2tj$, where distances are measured in centimetres. At $t = 0$ the particle is at the origin and has a velocity $v(t) = 2i + j$.

(a) Determine the velocity of the particle when $t = 2$ [2]

(b) Determine the position of the particle when it is moving parallel to the vertical axis [4]

(c) Explain why the particle can never move parallel to the horizontal axis [2]

(d) Sketch the path of the particle for $0 \le t \le 2$ [2]



(e) On the axes above sketch $v(0.5)$ [2]